

NCU-HEP-k016

Apr 2004

## On Extended Electroweak Symmetries.

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We discuss extensions of the Standard Model through extending the electroweak gauge symmetry. An extended electroweak symmetry requires a list of extra fermionic and scalar states. The former is necessary to maintain cancellation of gauge anomalies, and largely fixed by the symmetry embedding itself. The latter is usually considered quite arbitrary, so long as a vacuum structure admitting the symmetry breaking is allowed. Anomaly cancellation may be used to link the three families of quarks and leptons together, given a perspective on flavor physics. It is illustrated lately that the kind of models may also have the so-called little Higgs mechanism incorporated. This more or less fixes the scalar sector and take care of the hierarchy problem, making such models of extended electroweak symmetries quite appealing candidates as TeV scale effective field theories.

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\* Talk presented at CGC 2003 (Dec 4-17-21), Fort Lauderdale, FL USA  
— submission for the proceedings.

## ON EXTENDED ELECTROWEAK SYMMETRIES

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*Dedicated to Paul Frampton*

### 1. Introduction

This talk is my contribution to the event celebrating the 60th birthday of Paul Frampton. The subject here is extending electroweak symmetries, in particular, as an approach to particle physics beyond the Standard Model (SM). The focus is on my own works on the subject, which began during the time I was a student studying under Paul's supervision. I am getting back to the topic lately, with some studies more related to some of Paul's own works but in the name of little Higgs.

The SM is a model of interactions dictated by an  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge symmetry, with a anomaly free chiral fermion spectrum and a Higgs multiplet responsible for the spontaneous breaking of the electroweak (EW) symmetry  $SU(2)_L \times U(1)_Y$ . Extending the EW gauge symmetry

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\*Work partially supported by grant NSC 92-2112-M-008-044 of the National Science Council of Taiwan.

extends the SM while adding new fermions and scalars. This is to be contrasted with other approaches such as grand unification and/or supersymmetry. Compared with the latter approaches, extending EW symmetry may look like less popular or not so well motivated. Grand unification aims at providing a unified picture to all the otherwise separate parts of the gauge symmetry and their independent couplings, though only at a scale of about  $10^{-16}$  GeV. All that can be achieved, in the case of  $SU(5)$ , without the need for extra fermionic states. Supersymmetry used the beautiful boson-fermion symmetry to tackle the hierarchy problem, essentially extending the chiral nature of the fermions to fix the problem for the scalar sector. Putting the two together provides a theoretical structure that promises to “explain” more or less all of particle physics. However, the large extrapolation over the many order of magnitudes of particle physics desert may certainly be taken with suspicion. Moreover, the approaches do not provide any new insight into the difficult problem of the origin of flavor structure. Why there are three families of SM fermions is still a fundamental problem that we have no credible approach to handle. On the contrary, extending the EW symmetry may provide some new perspectives to the flavor problem. It can even provide an alternative solution to the hierarchy problem, in the name of the so-called little Higgs mechanism<sup>1</sup>.

## 2. Looking at the Fermionic Spectra

The spectrum of SM fermion in one family is like perfection, essentially dictated by gauge anomaly cancellation conditions. To illustrate the point of view, we recall our earlier argument<sup>2</sup>. Assuming that there exist a minimal multiplet carrying nontrivial quantum numbers of each of the component gauge groups, one can obtain the one-family SM spectrum as the unique solution by asking for the minimal consistent set of chiral states. Consistency here refers to the perfect cancellation of nonvanishing contributions to various gauge anomalies from individual fermionic states. A vectorlike set (or pair) is trivial but not interesting. Only chiral states are protected from heavy gauge invariant masses and relevant to physics at the relatively low energy scale.

The above suggested derivation of the one-family SM spectrum goes as follow. We are essentially starting with a quark doublet, with arbitrary hypercharge normalization. The two  $SU(3)_C$  triplets require two antitriplets to cancel the anomaly. Insisting on the chiral spectrum means taking two quark singlets here, with hypercharges still to be specified. Now,  $SU(2)_L$

is real, but has a global anomaly. Cancellation requires an even number of doublets, so at least one more beyond the three colored components in the quark doublet. There are still four anomaly cancellation conditions to take care of. They are the  $[SU(3)_C]^2 U(1)_Y$ ,  $[SU(2)_L]^2 U(1)_Y$ ,  $[grav]^2 U(1)_Y$ , and  $[U(1)_Y]^3$ . We are however left with three relative hypercharges to fit the four equations, actually without a possible solution. A rescue comes from simply adding a  $U(1)_Y$ -charged singlet. But the four equation for four unknown setting is misleading. The  $[U(1)_Y]^3$  anomaly cancellation equation is cubic in all the charges, with no rational solution guaranteed. The SM solution may actually be considered a beautiful surprise. Moreover, the perspective may be the best we have on understanding *why there is what there is*.

We would also like to take the opportunity here to briefly sketch the next step taken in Ref.<sup>2</sup>, to further illustrate our perspective. The results there also may be considered a worthy comparison with our little Higgs motivated flavor/family spectrum presented below, from the point of view of the origin of the three families. The major goal of Ref.<sup>2</sup> is to use a similar structure with an extended symmetry to obtain the three families. For example, one can start with some  $SU(4) \times SU(3) \times SU(2) \times U(1)$  gauge symmetry and try to obtain the minimal chiral spectrum contain a  $(\mathbf{4}, \mathbf{3}, \mathbf{2})$  multiplet — the simplest one with nontrivial quantum number under all component groups. Having a consistent solution is not enough though. In order for the spectrum be of interest, we ask the spectrum

multiplets	$X$	Gauge anomalies					$U(1)_Y$ states	
		trX	44X	33X	22X	$X^3$		
$(\mathbf{4}, \mathbf{3}, \mathbf{2})$	<b>1</b>	24	6	8	12	24	3 $\mathbf{1}(Q)$	<b>-5</b> ( $\bar{Q}'$ )
$(\bar{\mathbf{4}}, \mathbf{3}, \mathbf{1})$	<b>5</b>	60	15	20		1500	3 <b>-4</b> ( $\bar{u}$ )	<b>2</b> ( $\bar{d}$ )
$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	<b>3</b>	24	6		12	216	3 <b>-3</b> ( $L$ )	<b>3</b> ( $\bar{L}$ )
$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$	<b>9</b>	36	9			2916	3 <b>-6</b> ( $\bar{E}$ )	<b>0</b> ( $N$ )
$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	<b>-18</b>	-108	-36			-34992	3 <b>6</b> ( $E$ )	3 <b>12</b> ( $S$ )
$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{2})$	<b>-10</b>	-60		-20	-30	-6000	<b>5</b> ( $\bar{Q}'$ )	
$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$	<b>-4</b>	-12		-4		-192	<b>2</b> ( $\bar{d}$ )	
$(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})$	<b>-4</b>	-12		-4		-192	<b>2</b> ( $\bar{d}$ )	
$(\mathbf{1}, \mathbf{1}, \mathbf{2})$	<b>6</b>	12			6	432	<b>-3</b> ( $L$ )	
3 $(\mathbf{1}, \mathbf{1}, \mathbf{1})$	<b>24</b>	72				41472	3 <b>-12</b> ( $\bar{S}$ )	
3 $(\mathbf{1}, \mathbf{1}, \mathbf{1})$	<b>-12</b>	-36				-5184	3 <b>6</b> ( $E$ )	
<i>Total</i>		0	0	0	0	0		

Table I. An  $SU(4)_A \times SU(3)_C \times SU(2)_L \times U(1)_X$  spectrum embedding three SM families.

to yield the chiral spectrum of three SM families plus a set of vectorlike states under a feasible spontaneous symmetry breaking scenario, *i.e.* when the gauge symmetry is broken to that of the SM. Ref.<sup>2</sup> has only partial success. A consistent group theoretical SM embedding could be obtained but only with a slight addition to the minimal chiral spectrum obtained from anomaly cancellation considerations alone. We give an example in Table I.

Next, we recall the fermionic spectrum from a simple model of extended EW symmetry, the 331 model from Paul himself<sup>3</sup>. The model has the EW symmetry extended to an  $SU(3)_L \times U(1)_X$ . To have a consistent spectrum of chiral fermions, one may first look into how the SM doublets are to be embedded into multiplets of  $SU(3)_L$ . It is interesting to note here that a naive family universal embedding would not work. The  $SU(3)_L$  anomaly would not cancel. Instead, the model has the  $(t, b)$  doublet embedded into a  $\bar{\mathbf{3}}$  while the quark doublets of the first two families into  $\mathbf{3}$ 's, with all leptonic doublets embedded into  $\bar{\mathbf{3}}$ 's. The fact that the number of color equals the number of families makes the anomaly cancellation possible. All extra quark states here are exotic, with charges  $\frac{5}{3}$  and  $-\frac{4}{3}$ . There are no extra leptonic states though. The 331 model spectrum is given in Table II.

	$U(1)_Y$ -states	
$(\mathbf{3}_C, \bar{\mathbf{3}}_L, \frac{2}{3})$	$\frac{1}{6}[Q]$	$\frac{5}{3}(T)$
$2(\mathbf{3}_C, \mathbf{3}_L, \frac{-1}{3})$	$2\frac{1}{6}[2Q]$	$2\frac{-4}{3}(D, S)$
$3(\mathbf{1}_C, \bar{\mathbf{3}}_L, \mathbf{0})$	$3\frac{-1}{2}[3L]$	$3\mathbf{0}(3E^+)$
$3(\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{-2}{3})$	$4\frac{-2}{3}(\bar{u}, \bar{c}, \bar{t})$	
$3(\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{1}{3})$	$5\frac{1}{3}(\bar{d}, \bar{s}, \bar{b})$	
$1(\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{-5}{3})$	$\frac{-5}{3}(\bar{T})$	
$1(\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{4}{3})$	$2\frac{4}{3}(\bar{D}, \bar{S})$	

Table II. A (331) model spectrum for  $SU(3)_L \times U(1)_X$  extended EW symmetry.

### 3. Extended EW Symmetries of $SU(N)_L \times U(1)_X$

Looking at the model spectrum of Table II, one may wonder if the construction is in any sense unique, and if similar anomaly free spectra exist for a different extended EW symmetry. We look into the question lately and

have the general solution. It turns out quite simple and straightforward.

For an extended EW symmetry of  $SU(N)_L \times U(1)_X$ , the SM doublets may be embedded into  $\mathbf{N}$ 's or  $\bar{\mathbf{N}}$ 's. Embedding one quark doublet into an  $\mathbf{N}$  and the two others into  $\bar{\mathbf{N}}$ 's while putting all lepton doublets into  $\mathbf{N}$ 's does give a prescription with canceled  $SU(N)$  anomaly. The a bit of surprising part is that no matter how one chooses to embed  $U(1)_Y$  into  $SU(N)_L \times U(1)_X$ , simply completing the list of chiral states with appropriate  $SU(N)$  singlets to ensure vectorlike matchings at the QCD and QED level does yield a completely anomaly free spectrum, essential unique for the particular symmetry embedding. The number of possible consistent model spectra of the type is then equivalent to the number of admissible symmetry embeddings. The latter can conveniently be parametrized by the choice of electric charges for the extra  $N - 2$  quark states sharing the  $\mathbf{N}$  multiplet with the  $(t, b)$  doublet<sup>4</sup>. We have no room in this write-up to elaborate on the details though.

#### 4. Little Higgs and Extended Electroweak Symmetries

The little Higgs mechanism<sup>1</sup> has been proposed as new solution to the hierarchy problem. More precisely, it alleviates the quadratic divergent quantum correction to the SM Higgs states and admits a natural little hierarchy between the EW scale and a higher scale of so-called UV-completion at around 10 TeV above which further structure would be hidden. The idea is a rather humble bottom-up approach then; but experimental hints at the existence of such a little hierarchy has been discussed<sup>5</sup>. What is relevant for our present discussion is that a little Higgs model necessarily has an extended gauge symmetry, EW or beyond, and extra fermion(s). The latter includes a heavy top  $T$  quark.

Simple little Higgs model(s) based on an extended EW symmetry has been introduced by Kaplan and Schmaltz<sup>6</sup>, though the authors failed to properly address the structure of the fermionic sector. The gauge symmetry considered are  $SU(3)_L \times U(1)_X$  and  $SU(4)_L \times U(1)_X$ . We discuss completion of the kind of models with consistent, anomaly free, fermionic spectra and the resulted implications on the flavor structure of the models in Ref.<sup>4,7,8</sup>. Naively, so long as one pick a model spectrum with an extra  $T$  quark in the  $(t, b)$  containing  $\mathbf{N}$  multiplet (here  $N = 3$  or  $N = 4$ , for example), one have potentially a extended EW little Higgs model. The  $T$  quark may be used to cancel the quadratic divergent contributions (only at 1-loop level) to the SM Higgs mass from the  $t$  quark, while the extra EW gauge

bosons to do the same for their SM counterparts. The scalar/Higgs sector has to be explicitly constructed though, to have the SM Higgs doublet embedded as (pseudo-)Nambu-Goldstone states of some global symmetry. It is an  $[SU(3)]^2/[SU(2)]^2$  symmetry for the  $SU(3)_L \times U(1)_X$  case, for instance. The Higgs sector symmetry is to be explicitly violated beyond the sector, in the gauge and Yukawa couplings of the Higgs multiplets. Such a scheme can be easily achieved with pair(s) of Higgs multiplets having the right quantum number to couple to the  $(T, t, b, \dots)$  multiplet and a right-handed  $T$  singlet. However, there is source of further complication, related to the construction of a proper Higgs quartic coupling term<sup>6</sup>. We admit that, in general, the latter issue still have to be studied more carefully. We do have a definitely complete and consistent model though. This is given by the fermion spectrum of Table III, with the Higgs sector as given in Ref.<sup>6</sup>. Here below, we will focus on the fermionic sector and flavor physics structure.

With a specific choice of the extended EW symmetry, a little Higgs model can be built only with the inclusion of the  $T$  quark state. For the  $N = 3$  case, that fixes the hypercharge embedding and hence, from our anomaly cancellation study, the unique fermionic spectrum. The spectrum can be read off from Table III, with only one set of the duplicated  $T$ ,  $D$ ,  $S$ , and three  $N$  states. Note that the  $X$ -charges will have to change accordingly. For the  $N = 4$  case, one may consider variations of the model spectrum, essentially by choosing a different set of states beyond that of the  $N = 3$  content. In particular, a spectrum with a full set of duplicated, heavy, SM fermions look very interesting<sup>4</sup>. However, the scalar/Higgs sector has to be explicitly constructed then. Following exactly the construction of Ref.<sup>6</sup>, one may be restricted to the spectrum of Table III, with trivial

	Gauge anomalies					$U(1)_Y$ states	
	$\text{tr}X$	$LLL$	$LLX$	$CCX$	$_{(144)}X^3$		
$(\mathbf{3}_C, \mathbf{4}_L, \frac{5}{12})$	5	3	5/4	5/3	125	$\frac{1}{6}[Q]$	$2 \frac{2}{3}(T)$
$2(\mathbf{3}_C, \mathbf{4}_L, \frac{-1}{12})$	-2	-6	-1/2	-2/3	-2	$2 \frac{1}{6}[Q]$	$4 \frac{-1}{3}(2 D, 2 S)$
$3(\mathbf{1}_C, \mathbf{4}_L, \frac{-1}{4})$	-3	3	-3/4		-27	$3 \frac{-1}{2}[L]$	$6 \mathbf{0}(N)$
$5(\mathbf{3}_C, \mathbf{1}_L, \frac{-2}{3})$	-10			-10/3	-640	$5 \frac{-2}{3}$	$(\bar{u}, \bar{c}, \bar{t}, 2 \bar{T})$
$7(\mathbf{3}_C, \mathbf{1}_L, \frac{1}{3})$	7			7/3	112	$7 \frac{1}{3}$	$(d, \bar{s}, \bar{b}, 2 \bar{D}, 2 \bar{S})$
$3(\mathbf{1}_C, \mathbf{1}_L, \mathbf{1})$	3				432	$3 \mathbf{1}$	$(e^+, \mu^+, \tau^+)$
Total	0	0	0	0	0		

Table III. The  $SU(3)_C \times SU(4)_L \times U(1)_X$  chiral fermionic spectrum completing the Kaplan-Schmaltz little Higgs model, with anomaly cancellation illustrated.

generalization to  $N > 4$  spectra extending on the content. At the moment, one sees no motivation to go for  $N > 4$ .

### 5. Some Implications to Flavor Physics

Unlike generic models of extended EW symmetries, we do not have much freedom in picking a set of scalar multiplets with VEVs according to what mass generating Yukawa couplings we may want to include. However, a careful checking of the Higgs multiplets shows that phenomenologically acceptable mass terms for the fermions, SM ones or heavy quarks, can be obtained for the explicit models discussed above. Here, we use the  $SU(3)_L \times U(1)_X$  case for the demonstration, in favor of simpler notation and expressions.

As touched on above, the little Higgs mechanism is to be implemented with two scalar multiplets having the right quantum number to couple to the chiral parts of the  $T$  quark. They are denoted by  $\Phi_1$  and  $\Phi_2$  below in the expression of which we give the Yukawa part of the Lagrangian. The latter is constructed simply by tracing the quantum numbers and admitting all terms compatible with the gauge symmetries.

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & \lambda_1^t \bar{t}' \Phi_1 Q + \lambda_2^t \bar{T}' \Phi_2 Q + \frac{1}{M} \lambda_{\alpha j}^u \bar{u}'_\alpha \Phi_1 \Phi_2 Q'_j \\ & + \lambda_{\beta j}^{d1} \bar{d}'_\beta \Phi_1^\dagger Q'_j + \lambda_{\beta j}^{d2} \bar{d}'_\beta \Phi_2^\dagger Q'_j + \frac{1}{M} \lambda_\beta^b \bar{d}'_\beta \Phi_1^\dagger \Phi_2^\dagger Q, \quad (1) \end{aligned}$$

where  $Q$  and  $Q'_j$  denote (contrary to notation in Table III) the color triplet and antitriplets. Note that we have to include dimension five terms here. Recall that the little Higgs model actually has a high energy cut-off of only around a 10 TeV scale. The next step is to use the nonlinear sigma model expansion of the scalar multiplets in terms of the pseudo-Nambu-Goldstone states, which include the SM Higgs doublet  $h^{6,7}$ . We recover

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & f (\lambda_1^t \bar{t}' + \lambda_2^t \bar{T}') T + f (\lambda_{\beta j}^{d1} \bar{d}'_\beta + \lambda_{\beta j}^{d2} \bar{d}'_\beta) D_j \\ & + \frac{i}{\sqrt{2}} (\lambda_1^t \bar{t}' - \lambda_2^t \bar{T}') h \begin{pmatrix} t \\ b \end{pmatrix} - \frac{i\sqrt{2}f}{M} \lambda_{\alpha j}^u \bar{u}'_\alpha h \begin{pmatrix} u_j \\ d_j \end{pmatrix} \\ & - \frac{i}{\sqrt{2}} (\lambda_{\beta j}^{d1} \bar{d}'_\beta - \lambda_{\beta j}^{d2} \bar{d}'_\beta) h^\dagger \begin{pmatrix} u_j \\ d_j \end{pmatrix} + \frac{i\sqrt{2}f}{M} \lambda_\beta^b \bar{d}'_\beta h^\dagger \begin{pmatrix} t \\ b \end{pmatrix} + \dots \quad (2) \end{aligned}$$

The expression shows that all the heavy quark state,  $T$ , and  $D_j$  (or  $D$  and  $S$ ) get Dirac mass at scale  $f$  of the VEVs of  $\Phi_1$  and  $\Phi_2$ , and standard Yukawa couplings for the SM quarks and Higgs doublet are all available. However, the expression also indicates that one has to expect mass mixings among



heavy and SM quark states. The nature of the extra heavy quarks and their mass mixings with the SM counterparts dictate stringent constraints on the related couplings and interesting flavor physics.

## 6. Conclusions

The bottom line here is that sensible discussion of flavor physics of a little Higgs model is not possible before the full fermion spectrum is spelt out. The latter is constrained by gauge anomaly cancellation. We exhibit at least one complete model here on which detailed flavor physics still have to be studied. For the kind of models, the fermionic part has a family non-universal flavor structure just like that of the 331 model, linking the three SM families into one fully connected set. Gauge anomaly cancellation should play a major role on constructing the fermionic completion of any little Higgs model. This is, unfortunately, an issue that has been largely overlooked in the literature.

In summary, we see that studies of extended EW symmetries has arrived at the point of furnishing all round models of beyond SM physics addressing more or less all the concerns of particle physics, including the hierarchy problem. Such a model then has almost no arbitrary parts to be chosen at model-builders' discretion. It has generic appeals, but are also very humble, liable to various stringent precision EW and flavor physics constraints and begs UV-completion about an order of magnitude in energy scale above that of the electroweak theory. Building models of the kind, and studying their phenomenology in details, as well as checking the predictions experimentally should be a worthy endeavor.

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